Sétif 1 University
Faculty of Sciences
Common Core Mathematics and Computer Science

1st semester (year 2025/26)
Machine Structures 1 course
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Problem set 2 solution
(Number System)

Exercise 1:

1. Decimal to Binary:

- Convert $\mathbf{(25)_{10}}$ to binary using Successive Euclidean Division by 2 :

Division	Remainder
25 ÷ 2 = 12	1
12 ÷ 2 = 6	0
6 ÷ 2 = 3	0
3 ÷ 2 = 1	1
1 ÷ 2 = 0	1

$\mathbf{(25)_{10} = (11001)_2}$

- Convert $\mathbf{(100)_{10}}$ to binary using Successive Euclidean Division by 2:

Division	Remainder
100 ÷ 2 = 50	0
50 ÷ 2 = 25	0
25 ÷ 2 = 12	1
12 ÷ 2 = 6	0
6 ÷ 2 = 3	0
3 ÷ 2 = 1	1
1 ÷ 2 = 0	1

$\mathbf{(100)_{10} = (1100100)_2}$

- Convert $\mathbf{(153)_{10}}$ to binary using Successive Euclidean Division by 2:

Division	Remainder
153 ÷ 2 = 76	1
76 ÷ 2 = 38	0
38 ÷ 2 = 19	0
19 ÷ 2 = 9	1
9 ÷ 2 = 4	1
4 ÷ 2 = 2	0
2 ÷ 2 = 1	0
1 ÷ 2 = 0	1

$\mathbf{(153)_{10} = (10011001)_2}$

2. Binary to Decimal:

- Convert **(11011)**$_2$ to decimal using Polynomial Formula with base-2:

  $(1x2^4) + (1x2^3) + (0x2^2) + (1x2^1) + (1x2^0) = 16 + 8 + 0 + 2 + 1 =$ **(27)**$_{10}$

(11011)$_2$ **= (27)**$_{10}$

- Convert **(1000101)**$_2$ to decimal using Polynomial Formula with base-2:

  $(1x2^6) + (0x2^5) + (0x2^4) + (0x2^3) + (1x2^2) + (0x2^1) + (1x2^0)$
  $= 64 + 0 + 0 + 0 + 4 + 0 + 1 =$ **(69)**$_{10}$

(1000101)$_2$ **= (69)**$_{10}$

- Convert **(11111111)**$_2$ to decimal using Polynomial Formula with base-2:

  $(1x2^7) + (1x2^6) + (1x2^5) + (1x2^4) + (1x2^3) + (1x2^2) + (1x2^1) + (1x2^0)$
  $= 128 + 64 + 32 + 16 + 8 + 4 + 2 + 1 =$ **(255)**$_{10}$

(11111111)$_2$ **= (255)**$_{10}$

3. Decimal to Octal:

- Convert **(45)**$_{10}$ to octal using Successive Euclidean Division by 8:

Division	Remainder
45 ÷ 8 = 5	5
5 ÷ 8 = 0	5

(45)$_{10}$ **= (55)**$_8$

- Convert **(212)**$_{10}$ to octal using Successive Euclidean Division by 8:

Division	Remainder
212 ÷ 8 = 26	4
26 ÷ 8 = 3	2
3 ÷ 8 = 0	3

(212)$_{10}$ **= (324)**$_8$

4. Octal to Decimal:

- Convert **(63)**$_8$ to decimal using Polynomial Formula with base-8:

  $(6x8^1) + (3x8^0) = 48 + 3 =$ **(51)**$_{10}$

(63)$_8$ **= (51)**$_{10}$

- Convert **$(307)_8$** to decimal using Polynomial Formula with base-8:

  $(3x8^2) + (0x8^1) + (7x8^0) = (3x64) + 0 + 7 = 192 + 7 =$ **$(199)_{10}$**

$(307)_8 = (199)_{10}$

5. Decimal to Hexadecimal:

- Convert **$(30)_{10}$** to hexadecimal using Successive Euclidean Division by 16:

Division	Remainder
30 ÷ 16 = 1	14 (E)
1 ÷ 16 = 0	1

$(30)_{10} = (1E)_{16}$

- Convert **$(4096)_{10}$** to hexadecimal using Successive Euclidean Division by 16:

Division	Remainder
4096 ÷ 16 = 256	0
256 ÷ 16 = 16	0
16 ÷ 16 = 1	0
1 ÷ 16 = 0	1

$(4096)_{10} = (1000)_{16}$

6. Hexadecimal to Decimal:

- Convert **$(1A)_{16}$** to decimal using Polynomial Formula with base-16:

  $(1x16^1) + (Ax16^0) = (1x16) + (10x1) = 16 + 10 =$ **$(26)_{10}$**

$(1A)_{16} = (26)_{10}$

- Convert **$(C3D)_{16}$** to decimal using Polynomial Formula with base-16:

  $(Cx16^2) + (3x16^1) + (Dx16^0) = (12x256) + (3x16) + (13x1)$
  $= 3072 + 48 + 13 =$ **$(3133)_{10}$**

$(C3D)_{16} = (3133)_{10}$

7. Binary to Octal/Hexadecimal:

- Convert **$(101101010)_2$** to octal using 3-bits equivalence method:

  $(101 \mid 101 \mid 010)_2 = (5 \mid 5 \mid 2)_8$

$(101101010)_2 = (552)_8$

- Convert **$(11100101100)_2$** to hexadecimal using 4-bits equivalence method:

  $(0111 \mid 0010 \mid 1100)_2 = (7 \mid 2 \mid C)_{16}$

$(11100101100)_2 = (72C)_{16}$

8. Octal/Hexadecimal to Binary:

- Convert **$(725)_8$** to binary using 3-bits equivalence method:

  $(7\ 2\ 5)_8 = (111 \mid 010 \mid 101)_2$

$(725)_8 = (111010101)_2$

- Convert **$(A5B)_{16}$** to binary using 4-bits equivalence method:

  $(A\ 5\ B)_{16} = (1010 \mid 0101 \mid 1011)_2$

$(A5B)_{16} = (101001011011)_2$

9. Octal to Hexadecimal:

- Convert **$(724)_8$** to hexadecimal going through binary:

  $(7\ 2\ 4)_8 = (111 \mid 010 \mid 100)_2 = (0001 \mid 1101 \mid 0100)_2 = \mathbf{(1\ D\ 4)_{16}}$

$(724)_8 = (1D4)_{16}$

- Convert **$(3051)_8$** to hexadecimal going through binary:

  $(3\ 0\ 5\ 1)_8 = (011 \mid 000 \mid 101 \mid 001)_2 = (0110 \mid 0010 \mid 1001)_2 = \mathbf{(6\ 2\ 9)_{16}}$

$(3051)_8 = (629)_{16}$

10. Hexadecimal to Octal:

- Convert **$(A6)_{16}$** to octal going through binary:

  $(A\ 6)_{16} = (1010 \mid 0110)_2 = (010 \mid 100 \mid 110)_2 = \mathbf{(2\ 4\ 6)_8}$

$(A6)_{16} = (246)_8$

- Convert **$(1D3)_{16}$** to octal going through binary:

  $(1\ D\ 3)_{16} = (0001 \mid 1101 \mid 0011)_2 = (000 \mid 111 \mid 010 \mid 011)_2 = \mathbf{(0\ 7\ 2\ 3)_8}$

$(1D3)_{16} = (723)_8$

Exercise 2:

1. It is possible to write the number 4000_{10} in sexagesimal by converting 4000 to base-60**:**

Division	Remainder
4000 ÷ 60 = 66	40
66 ÷ 60 = 1	6
1 ÷ 60 = 0	1

So, **$(4000)_{10}$** in base 60 is represented as **$(1, 6, 40)_{60}$.**

<u>Remark 1</u>: It is important to understand that 40 is not a number but the digit 40.

2. From the previous answer in **4000 seconds** we have: **1 hour : 6 minutes : 40 seconds**

Exercise 3:

1. Determining the unknown base *x* solving the following equation $(101)_x = (26)_{10}$

To solve this, we need to convert $(101)_x$ to base-10.

$$(101)_x = 1 \cdot x^2 + 0 \cdot x^1 + 1 \cdot x^0$$
$$(101)_x = x^2 + 0 + 1$$
$$(101)_x = x^2 + 1$$

Now we set this equal to $(26)_{10}$:

$$x^2 + 1 = 26$$
$$x^2 = 26 - 1$$
$$x^2 = 25$$
$$x = \sqrt{25}$$
$$x = 5 \quad \text{(Since bases are positive)}$$

So, the unknown base **x = 5**.

2. Once x is found, convert the number $(142)_x$ to Decimal, Binary, and Hexadecimal.

Now that x = 5, we need to convert $(142)_5$ to Decimal, Binary, and Hexadecimal:

a. $(142)_5$ to Decimal using Polynomial Formula with base-5:

$$\mathbf{(142)_5} = 1 \cdot 5^2 + 4 \cdot 5^1 + 2 \cdot 5^0$$
$$= 1 \cdot 25 + 4 \cdot 5 + 2 \cdot 1$$
$$= 25 + 20 + 2$$
$$= \mathbf{(47)_{10}}$$

b. $(142)_5$ (which is $(47)_{10}$) to Binary:
We'll convert $(47)_{10}$ to binary using Successive Euclidean Division by 2:

Division	Remainder
47 ÷ 2 = 23	1
23 ÷ 2 = 11	1
11 ÷ 2 = 5	1
5 ÷ 2 = 2	1
2 ÷ 2 = 1	0
1 ÷ 2 = 0	1

$(47)_{10}$ = $(101111)_2$

c. $(142)_5$ (which is $(101111)_2$) to Hexadecimal:

We'll convert $(101111)_2$ to hexadecimal using 4-bits equivalence method:

$(101111)_2 = (0010 \mid 1111)_2 = \mathbf{(2F)_{16}}$

$(47)_{10}$ = $(2F)_{16}$

3. Convert the decimal number $(205)_{10}$ to base x (which is base-5).

We'll convert $(205)_{10}$ to base-5 using Successive Euclidean Division by 5:

Division	Remainder
205 / 5 = 41	0
41 / 5 = 8	1
8 / 5 = 1	3
1 / 5 = 0	1

$(205)_{10}$ = $(1310)_5$

Exercise 4:

1. Decimal Fraction to Binary Fraction conversion:

- Convert **$(0.75)_{10}$** to binary fraction using Successive Multiplication by 2:

Multiplication	Integer part
0.75 x 2 = 1.50	1
0.50 x 2 = 1.00	1

$(0.75)_{10}$ = $(0.11)_2$

- Convert **$(0.125)_{10}$** to binary fraction using Successive Multiplication by 2:

Multiplication	Integer part
0.125 x 2 = 0.25	0
0.25 x 2 = 0.50	0
0.50 x 2 = 1.00	1

$(0.125)_{10} = (0.001)_2$

- Convert **$(12.8125)_{10}$** to binary:

First, convert the integer part $(12)_{10}$ to binary using Successive Euclidean Division by 2:

Division	Remainder
12 ÷ 2 = 6	0
6 ÷ 2 = 3	0
3 ÷ 2 = 1	1
1 ÷ 2 = 0	1

$(12)_{10} = (1100)_2$

Next, convert the fractional part $(0.8125)_{10}$ to binary fraction using Successive Multiplication by 2:

Multiplication	Integer part
0.8125 x 2 = 1.6250	1
0.6250 x 2 = 1.2500	1
0.2500 x 2 = 0.5000	0
0.5000 x 2 = 1.0000	1

$(0.8125)_{10} = (0.1101)_2$

Combine the integer and fractional parts: **$(12.8125)_{10} = (1100.1101)_2$**

2. Binary Fraction to Decimal Fraction conversion:

- Convert **$(0.11)_2$** to decimal fraction using Fractional Polynomial Formula with base-2:

$$
\begin{aligned}
(0.11)_2 &= 1 \times 2^{-1} + 1 \times 2^{-2} \\
&= 1 \times (1/2) + 1 \times (1/4) \\
&= 0.5 + 0.25 \\
&= 0.75
\end{aligned}
$$

$(0.11)_2 = (0.75)_{10}$

- Convert **$(0.0101)_2$** to decimal fraction using Fractional Polynomial Formula with base-2:

$$
\begin{aligned}
(0.0101)_2 &= 0 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} \\
&= 0 + 1 \times (1/4) + 0 + 1 \times (1/16)
\end{aligned}
$$

$$= 0.25 + 0.0625$$
$$= 0.3125$$

$(0.0101)_2 = (0.3125)_{10}$

- Convert **$(1101.1011)_2$** to decimal:

First, convert the integer part $(1101)_2$ to decimal using Polynomial Formula with base-2:

$$\begin{aligned}(1101)_2 &= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 \\ &= 1 \times 8 + 1 \times 4 + 0 \times 2 + 1 \times 1 \\ &= 8 + 4 + 0 + 1 \\ &= 13\end{aligned}$$

$(1101)_2 = (13)_{10}$

Next, convert the fractional part $(0.1011)_2$ to decimal using Fractional Polynomial Formula with base-2:

$$\begin{aligned}(0.1011)_2 &= 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\ &= 1 \times (1/2) + 0 + 1 \times (1/8) + 1 \times (1/16) \\ &= 0.5 + 0.125 + 0.0625 \\ &= 0.6875\end{aligned}$$

$(0.1011)_2 = (0.6875)_{10}$

Combine the integer and fractional parts: **$(1101.1011)_2 = (13.6875)_{10}$**

3. Decimal Fraction to Octal Fraction conversion:

- Convert **$(0.5)_{10}$** to octal fraction using Successive Multiplication by 8:

Multiplication	Integer part
0.5 x 8 = 4.0	4

$(0.5)_{10} = (0.4)_8$

- Convert **$(87.25)_{10}$** to octal:

First, convert the integer part $(87)_{10}$ to octal using Successive Euclidean Division by 8:

Multiplication	Integer part
87 ÷ 8 = 10	7
10 ÷ 8 = 1	2
1 ÷ 8 = 0	1

$(87)_{10} = (127)_8$

Next, convert the fractional part $(0.25)_{10}$ to octal fraction using Successive Multiplication by 8:

Multiplication	Integer part
0.25 x 8 = 2.0	2

$(0.25)_{10}$ = $(0.2)_8$

Combine the integer and fractional parts: **$(87.25)_{10}$ = $(127.2)_8$**

4. Octal Fraction to Decimal Fraction conversion using Fractional Polynomial Formula with base-8:

- Convert **$(0.4)_8$** to decimal fraction.

  $(0.4)_8 = 4 \times 8^{-1} = 4 \times (1/8) = 4/8 = 0.5$

  $(0.4)_8$ = $(0.5)_{10}$

- Convert **$(156.12)_8$** to decimal:

First, convert the integer part $(156)_8$ to decimal using Polynomial Formula with base-8:

$$\begin{aligned}(156)_8 &= 1 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 \\ &= 1 \times 64 + 5 \times 8 + 6 \times 1 \\ &= 64 + 40 + 6 \\ &= 110\end{aligned}$$

$(156)_8$ = $(110)_{10}$

Next, convert the fractional part $(0.12)_8$ to decimal using Fractional Polynomial Formula with base-8:

$$\begin{aligned}(0.12)_8 &= 1 \times 8^{-1} + 2 \times 8^{-2} \\ &= 1 \times (1/8) + 2 \times (1/64) \\ &= 0.125 + 2 \times 0.015625 \\ &= 0.125 + 0.03125 \\ &= 0.15625\end{aligned}$$

$(0.12)_8$ = $(0.15625)_{10}$

Combine the integer and fractional parts: **$(156.12)_8$ = $(110.15625)_{10}$**

5. Decimal Fraction to Hexadecimal conversion:

- Convert **$(0.5)_{10}$** to hexadecimal using Successive Multiplication by 16:

Multiplication	Integer part
0.5 x 16 = 8.0	8

$(0.5)_{10}$ = $(0.8)_{16}$

- Convert **$(257.0625)_{10}$** to hexadecimal:

First, convert the integer part $(257)_{10}$ to hexadecimal using Successive Euclidean Division by 16:

Division	Remainder
257 ÷ 16 = 16	1
16 ÷ 16 = 1	0
1 ÷ 16 = 0	1

$(257)_{10} = (101)_{16}$

Next, convert the fractional part $(0.0625)_{10}$ to hexadecimal using Successive Multiplication by 16:

Multiplication	Integer part
0.0625 x 16 = 1.0	1

$(0.0625)_{10} = (0.1)_{16}$

Combine the integer and fractional parts: **$(257.0625)_{10} = (101.1)_{16}$**

6. Hexadecimal Fraction to Decimal conversion:

- Convert **$(0.8)_{16}$** to decimal fraction.

$(0.8)_{16} = 8 \times 16^{-1} = 8 \times (1/16) = 8/16 = 0.5$

$(0.8)_{16} = (0.5)_{10}$

- Convert **$(D5.1C)_{16}$** to decimal.

First, convert the integer part $(D5)_{16}$ to decimal using Polynomial Formula with base-16:

$$\begin{aligned}(D5)_{16} &= 13 \times 16^1 + 5 \times 16^0 \\ &= 13 \times 16 + 5 \times 1 \\ &= 208 + 5 \\ &= 213\end{aligned}$$

$(D5)_{16} = (213)_{10}$

Next, convert the fractional part $(0.1C)_{16}$ to decimal using Fractional Polynomial Formula with base-16:

$$\begin{aligned}(0.1C)_{16} &= 1 \times 16^{-1} + 12 \times 16^{-2} \\ &= 1 \times (1/16) + 12 \times (1/256) \\ &= 0.0625 + 12 \times 0.00390625 \\ &= 0.0625 + 0.046875 \\ &= 0.109375\end{aligned}$$

$(0.1C)_{16} = (0.109375)_{10}$

Combine the integer and fractional parts: **$(D5.1C)_{16} = (213.109375)_{10}$**

Exercise 5:

1. Binary Addition:

$(1011)_2$ + $(0010)_2$ = $(1101)_2$

```
      1
    1 0 1 1  ←11
  + 0 0 1 0  ← 2
  ---------
  = 1 1 0 1  ←13
```

$(1110)_2$ + $(0101)_2$ = $(10011)_2$

```
     1
   1 1 1 1 0  ←14
 +   0 1 0 1  ← 5
 -----------
 = 1 0 0 1 1  ←19
```

$(100.11)_2$ + $(110.1)_2$ = $(1011.01)_2$

Align the binary points and pad with zeros:

```
          1
   1 1 0 0.1 1  ←  4.75
 +   1 1 0.1 0  ←  6.5
 -------------
 = 1 0 1 1.0 1  ← 11.25
```

2. Binary Subtraction:

$(1101)_2$ - $(0100)_2$ = $(1001)_2$

```
    1 1 0 1  ← 13
  - 0 1 0 0  ←  4
  ---------
  = 1 0 0 1  ←  9
```

$(1000)_2$ + $(-0011)_2$ = $(0101)_2$

This is equivalent to 1000_2 - 0011_2:

```
    1 ₁0 ₁0 ₁0  ← 8
  - 0  0  1  1  ← 3
       1  1  1
  ------------
  = 0  1  0  1  ← 5
```

$(0100)_2$ - $(1011)_2$ = $(-111)_2$

Like in decimal, when the subtracted value is greater that the minuend (first value), we reverse the subtraction and minus the result.

```
    1 ₁0 1 1  ← 11
  - 0  1 0 0  ←  4
    1
  ----------
  = 0  1 1 1  ←  7
```

$(10.110)_2$ - $(10.1)_2$ = $(0.01)_2$

Align the binary points and pad zeros:

$$\begin{array}{rl}
10.110 & \leftarrow 2.75 \\
-\ 10.100 & \leftarrow 2.25 \\
\hline
=\ 00.010 & \leftarrow 0.25
\end{array}$$

3. Binary Multiplication:

$(101)_2$ × $(10)_2$ = $(1010)_2$

$$\begin{array}{rl}
101 & \leftarrow 5 \\
\times\ \ 10 & \leftarrow 2 \\
\hline
=\ 000 & \\
+\ 101\cdot & \\
\hline
=\ 1010 & \leftarrow 10
\end{array}$$

$(110)_2$ × $(11)_2$ = $(10010)_2$

$$\begin{array}{rl}
110 & \leftarrow 6 \\
\times\ \ 11 & \leftarrow 3 \\
\hline
=\ 110 & \\
{}^{1\ 1} & \\
+\ 110\cdot & \\
\hline
=\ 10010 & \leftarrow 18
\end{array}$$

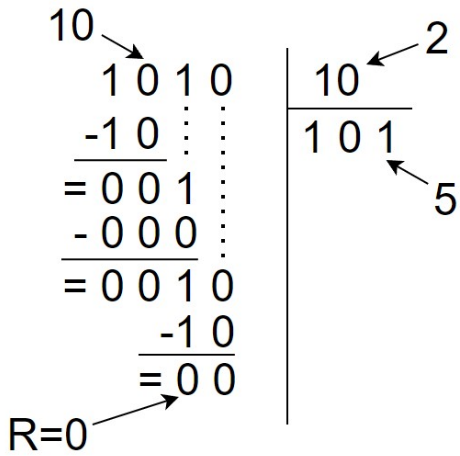
$(10.11)_2$ × $(-10.1)_2$ = $(-110.111)_2$

Like decimal, we ignore the negative sign and fractal dot. Then we multiply $(1011)_2$ by $(101)_2$. The final answer will be negative, and the fractal dot is at position 3 (2+1):

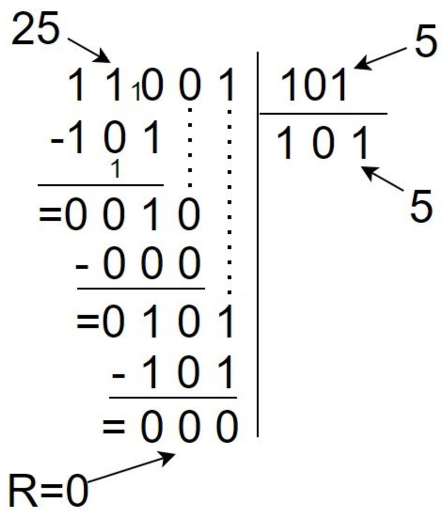
$$\begin{array}{rl}
1011 & \leftarrow 11 \\
\times\ 101 & \leftarrow 5 \\
\hline
=\ 1011 & \\
{}^{1}\ \ \ \ \ \ & \\
+\ 0000\cdot & \\
+\ 1011\cdot\cdot & \\
\hline
=\ 110111 & \leftarrow 55
\end{array}$$

4. Binary Division:

$(1010)_2 \div (10)_2 = (101)_2$ | R = $(0)_2$



$(11001)_2 \div (101)_2 = (101)_2$ | R = $(0)_2$

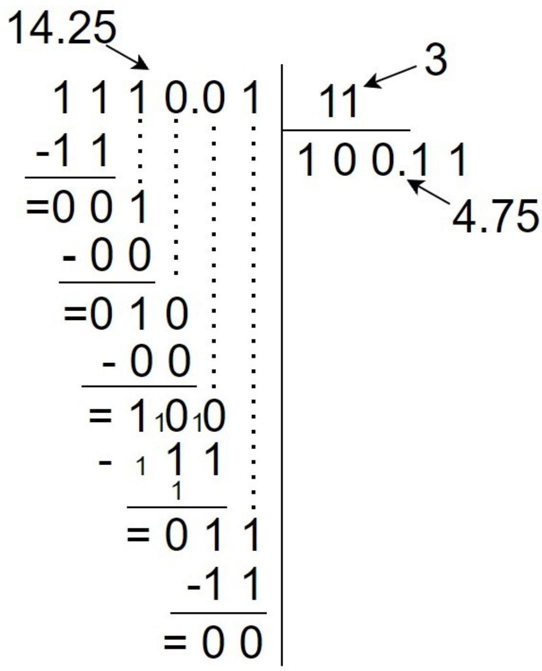


$(-111.001)_2 \div (1.10)_2 = (-100.11)_2$

First, we ignore the negative sign, the result will be negative. Like decimal, we need to make the divisor an integer by shifting the binary point of both numbers. The divisor $(1.1)_2$ has 1 fractional digits, so we shift both by 1 place to the right. The dividend $(111.001)_2$ becomes $(1110.01)_2$ and the divisor $(1.10)_2$ becomes $(11)_2$.

We get then do $(1110.01)_2 \div (11)_2$ like a real division and not integer division:

Remark 2: We clearly distinguish between the integer division (with remainder), and real division with fractional dot. It is always possible to perform a real division like it was done in decimal base.



5. The calculation is similar to decimal while multiplying by 10^n:

- **$(1011)_2 \times 2^2 = (1011)_2 \times (100)_2 = (101100)_2$**
- **$(110)_2 \times (1000)_2 = (110)_2 \times 2^3 = (110000)_2$**
- **$(1100.101)_2 \times 2^4 = (1100.101)_2 \times (10000)_2 = (11001010)_2$**

6. The calculation is similar to decimal while dividing by 10^n:

- **$(11000)_2 \div 2^3 = (11000)_2 \div (1000)_2 = (11)_2$**
- **$(10111)_2 \div (1000)_2 = (10111)_2 \div 2^3 = (10.111)_2$**
- **$(110100.101)_2 \div 2^4 = (110100.101)_2 \div (10000)_2 = (11.0100101)_2$**

Q1: Let consider the powers of 2 in binary:

$2^0 = (1)_2$
$2^1 = (10)_2$
$2^2 = (100)_2$
$2^3 = (1000)_2$
$2^4 = (10000)_2$
$2^5 = (100000)_2$

From this pattern, we can conclude that the binary representation of:
2^n is simply a 1 followed by *n* zeros.

We can easily generalize the formula to any base, it is exactly the same:
Base^n is simply a 1 followed by *n* zeros. <u>Ex</u>**:** $8^3 = (1000)_8$

Q2: From 5 solutions, we can conclude that multiplying a binary number by 2^n is equivalent to shifting the binary dot *n* places to the right, or shifting the entire number if it's an integer *n* places to the left, padding zeros in LSB.

And from 6 solutions, we can conclude that dividing a binary number by 2^n is equivalent to shifting the binary dot *n* places to the left, or shifting the entire number if it's an integer *n* places to the right, effectively removing *n* zeros from LSB.

We can generalize this observation to any base b. If you multiply a number in base b by b^n, you shift the radix dot *n* positions to the right, or shifting the number left by padding zeros. And if you divide a number in base b by b^n, you shift the radix point *n* positions to the left, or shifting the entire number to the right by erasing zeros.