Sétif 1 University
Faculty of Sciences
Common Core Mathematics and Computer Science

1st semester (year 2025/26)
Machine Structures 1 course
Kara Abdelaziz professor

Problem set 4 solution (Boolean Algebra)

Exercise 1:

1). The Truth Table of the function: $F(A,B) = AB+A\overline{B}$

A	B	$AB+A\overline{B}$
0	0	0
0	1	0
1	0	1
1	1	1

The Truth Table of the function: $F(A,B) = (\overline{A+B})+A$

A	B	$\overline{A+B}$	$(\overline{A+B})+A$
0	0	1	1
0	1	0	0
1	0	0	1
1	1	0	1

2). The Truth Table of the function: $F(A,B,C) = ABC+A\overline{B}C+\overline{A}BC$

A	B	C	ABC	$A\overline{B}C$	$\overline{A}BC$	$ABC+A\overline{B}C+\overline{A}BC$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	1
1	0	0	0	0	0	0
1	0	1	0	1	0	1
1	1	0	0	0	0	0
1	1	1	1	0	0	1

The Truth Table of the function: $F(A,B,C) = (A+B)(A+\overline{C})$

A	B	C	$A+B$	$A+\overline{C}$	$(A+B)(A+\overline{C})$
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	1
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	1	1	1
1	1	0	1	1	1
1	1	1	1	1	1

3). The Truth Table of the function: $F(A,B,C,D) = \overline{(\overline{A+B})(\overline{C+D})}$

A	B	C	D	$\overline{A+B}$	$\overline{C+D}$	$\overline{(\overline{A+B})(\overline{C+D})}$
0	0	0	0	1	1	0
0	0	0	1	1	0	1
0	0	1	0	1	0	1
0	0	1	1	1	0	1
0	1	0	0	0	1	1
0	1	0	1	0	0	1
0	1	1	0	0	0	1
0	1	1	1	0	0	1
1	0	0	0	0	1	1
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	0	1
1	1	0	0	0	1	1
1	1	0	1	0	0	1
1	1	1	0	0	0	1
1	1	1	1	0	0	1

The Truth Table of the function: $F(A,B,C,D) = \bar{A}\bar{B}CD+\bar{A}BCD+A\bar{B}CD+ABCD$

A	B	C	D	$\bar{A}\bar{B}CD$	$\bar{A}BCD$	$A\bar{B}CD$	$ABCD$	$F(A,B,C,D)$
0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	1	1	1	0	0	0	1
0	1	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0
0	1	1	0	0	0	0	0	0
0	1	1	1	0	1	0	0	1
1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	0	0	0	0	0
1	0	1	1	0	0	1	0	1
1	1	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	0
1	1	1	0	0	0	0	0	0
1	1	1	1	0	0	0	1	1

Exercise 2:

1. The SoP form of the Truth Table is: $F(A,B) = (\bar{A}B) + (AB)$

The PoS form of the Truth Table is: $F(A,B) = (A+B)(\bar{A}+B)$

2. The SoP form of the Truth Table is:

$$F(A,B,C) = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}C + AB\bar{C}$$

The PoS form of the Truth Table is:

$$F(A,B,C) = (A+\bar{B}+C)(A+\bar{B}+\bar{C})(\bar{A}+B+C)(\bar{A}+\bar{B}+\bar{C})$$

3. The SoP form of the Truth Table is:

$$F(A,B,C,D) = \bar{A}\bar{B}\bar{C}D + \bar{A}B\overline{CD} + \bar{A}B\bar{C}D + \bar{A}BC\bar{D} + A\bar{B}CD + AB\bar{C}D + ABC\bar{D}$$

The PoS form of the Truth Table is:

$$F(A,B,C,D) = (A+B+C+D)(A+B+\bar{C}+D)(A+B+\bar{C}+\bar{D})(A+\bar{B}+\bar{C}+\bar{D})(\bar{A}+B+C+D)(\bar{A}+B+C+\bar{D})$$
$$(\bar{A}+B+\bar{C}+D)(\bar{A}+\bar{B}+C+D)(\bar{A}+\bar{B}+\bar{C}+\bar{D})$$

Exercise 3:

1. Reduction of the Boolean expression: $AC + \bar{A}\bar{B}C$

Expression	Theorems
$AC + \bar{A}\bar{B}C$	
$A(1)C + \bar{A}\bar{B}C$	T1
$A(B+\bar{B})C + \bar{A}\bar{B}C$	T5’
$ABC + A\bar{B}C + \bar{A}\bar{B}C$	T8
$(ABC + A\bar{B}C) + (A\bar{B}C + \bar{A}\bar{B}C)$	T3’
$AC + \bar{B}C$	T10
$(A+\bar{B})C$	T8

Remark 1: Student does not need to show the theorem used for simplification. They are put here just for clarification.

2. Reduction of the Boolean expression: $\bar{A}\bar{B} + \bar{A}B\bar{C} + \overline{(A + \bar{C})}$

Expression	Theorems
$\bar{A}\bar{B} + \bar{A}B\bar{C} + \overline{(A+\bar{C})}$	
$\bar{A}\bar{B} + \bar{A}B\bar{C} + (\bar{A}\cdot\bar{\bar{C}})$	T11’
$\bar{A}\bar{B} + \bar{A}B\bar{C} + \bar{A}C$	T4
$\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$	T10
$\bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC$	T3’
$\bar{A}\bar{B}C + \bar{A}BC + \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C}$	T6’
$\bar{A}C + \bar{A}\bar{C}$	T10
$\bar{A}$	T10

3. Reduction of the Boolean expression: $(\overline{A+B+C})D + AD + B$

Expression	Theorems
$(\overline{A+B+C})D + AD + B$	
$(\bar{A}\cdot\bar{B}\cdot\bar{C})D + AD + B$	T11'
$(\bar{A}\bar{B}\bar{C}+A)D + B$	T8
$(\bar{A}\bar{B}\bar{C}+A\bar{B}\bar{C}+A\bar{\bar{B}}\bar{\bar{C}})D + B$	T10
$((\bar{A}\bar{B}\bar{C}+A\bar{B}\bar{C})+(A\bar{B}\bar{C}+A\bar{\bar{B}}\bar{\bar{C}}))D + B$	T3'
$(\bar{B}\bar{C}+A)D + B$	T10
$\bar{B}\bar{C}D + AD + B$	T8
$(\bar{B}\bar{C}D+B) + AD$	T6'
$(\bar{B}\bar{C}D+B\bar{C}D+B\bar{\bar{C}}D) + AD$	T10
$((\bar{B}\bar{C}D+B\bar{C}D)+(B\bar{C}D+B\bar{\bar{C}}D)) + AD$	T3'
$\bar{C}D+B+AD$	T10

Remark 2: By observing the previous problem, we can formulate this theorem: $X+\bar{X}Y=X+Y$ It was used a lot in this problem. Its dual $X(\bar{X}+Y)=XY$ is easily verifiable correct. Those 2 theorems are considered valid for exams.

4. Reduction of the Boolean expression: $AB + (\overline{A+B}) + (A+B)(\bar{A}+\bar{B})$

Expression	Theorems
$AB + (\overline{A+B}) + (A+B)(\bar{A}+\bar{B})$	
$AB + (\bar{A}\cdot\bar{B}) + (A+B)(\bar{A}+\bar{B})$	T11'
$A\ominus B + (A+B)(\bar{A}+\bar{B})$	Xnor
$A\ominus B + (A(\bar{A}+\bar{B})+B(\bar{A}+\bar{B}))$	T8
$A\ominus B + ((A\bar{A}+A\bar{B})+(B\bar{A}+B\bar{B}))$	T8
$A\ominus B + ((0+A\bar{B})+(B\bar{A}+0))$	T5
$A\ominus B + (A\bar{B}+B\bar{A})$	T1'
$A\ominus B + (A\bar{B}+\bar{A}B)$	T6
$A\ominus B + A\oplus B$	Xor

Exercise 4:

1). Karnaugh map method using SoP (Minterms) and PoS (Maxterms):

1.

A	B	C	F(A,B,C)
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

C \ AB	00	01	11	10
0	1	1	1	0
1	0	1	0	0

C \ AB	00	01	11	10
0	1	1	1	0
1	0	1	0	0

The SoP reduction is: $F(A,B,C) = \bar{A}\bar{C} + B\bar{C} + \bar{A}B$

The PoS reduction is: $F(A,B,C) = (\bar{A}+\bar{C})(B+\bar{C})(\bar{A}+B)$

2.

A	B	C	F(A,B,C)
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

C \ AB	00	01	11	10
0	0	1	1	0
1	1	0	1	1

C \ AB	00	01	11	10
0	0	1	1	0
1	1	0	1	1

The SoP reduction is: $F(A,B,C) = B\overline{C} + AC + \overline{B}C$

The PoS reduction is: $F(A,B,C) = (B+C)(A+\overline{B}+\overline{C})$

3.

A	B	C	D	F(A,B,C,D)
0	0	0	0	1
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	1	0
11	1	0	0	1
10	0	1	1	0

CD \ AB	00	01	11	10
00	1	0	0	1
01	1	0	1	0
11	1	0	0	1
10	0	1	1	0

The SoP reduction is: $F(A,B,C,D) = BC\overline{D} + \overline{B}\overline{C}\overline{D} + \overline{B}CD + \overline{A}\overline{B}D + AB\overline{C}D$

The PoS reduction is: $F(A,B,C,D) = (\overline{B}+C+D)(\overline{B}+\overline{C}+\overline{D})(B+\overline{C}+D)(A+\overline{B}+C)(\overline{A}+B+C+\overline{D})$

4.

A	B	C	D	F(A,B,C,D)
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	0	0	1
11	1	0	1	0
10	1	1	0	0

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	0	0	1
11	1	0	1	0
10	1	1	0	0

The SoP reduction is: $F(A,B,C,D) = B\overline{C}\overline{D} + \overline{A}C\overline{D} + \overline{A}\overline{B}C + A\overline{B}\overline{C} + ABCD$

The PoS reduction is: $F(A,B,C,D) = (\overline{B}+C+\overline{D})(\overline{A}+\overline{C}+D)(A+B+C)(A+\overline{B}+\overline{D})(\overline{A}+B+\overline{C})$

2). Quine-MacCluskey method:

1. Stage 1:

Minterms	Pass 1
(0) 000 ✓	(2,0) 0-0 ✗
(2) 010 ✓	(2,3) 01- ✗
(3) 011 ✓	(2,6) -10 ✗
(6) 110 ✓	

Stage 2:

Implicant \ Minterm	(0) 000	(2) 010	(3) 011	(6) 110
(2,0) 0-0	x	x		
(2,3) 01-		x	x	
(2,6) -10		x		x

EPI = {(2,0)0-0, (2,3)01- , (2,6)-10}.
$\text{EPI}_{\text{Minterm}}$ = {(0)000, (2)010, (3)011, (6)110}. all Minterms are covered.

$F(A,B,C) = \overline{A}\overline{C} + \overline{A}B + B\overline{C}$

2. Stage 1:

Minterms	Pass 1
(1) 001 ✓	(1,5) -01 ✗
(2) 010 ✓	(2,6) -10 ✗
(5) 101 ✓	(5,7) 1-1 ✗
(6) 110 ✓	(6,7) 11- ✗
(7) 111 ✓	

Stage 2:

Implicant \ Minterm	(1) 001	(2) 010	(5) 101	(6) 110	(7) 111
(1,5) -01	x		x		
(2,6) -10		x		x	
(5,7) 1-1			x		x
(6,7) 11-				x	x

EPI = {(1,5)-01, (2,6)-10}.
$\text{EPI}_{\text{Minterm}}$ = {(1)001, (2)010, (5)101, (6)110}. Not all Minterms are covered.
$\overline{\text{EPI}}_{\text{Minterm}}$ = {(7)111}
NEPI = {(5,7)1-1, (6,7)11-}.

Either of the two NEPI are valid to cover $\overline{\text{EPI}}_{\text{Minterm}}$, we can choose one of them.

The result is (5,7)1-1 and EPI: $F(A,B,C) = \overline{B}C + B\overline{C} + AC$

3. Stage 1:

Minterms	Pass 1
(0) 0000✓	(0,1) 000-✗
(1) 0001✓	(0,8) -000✗
(8) 1000✓	(1,3) 00-1✗
(3) 0011✓	(3,11) -011✗
(6) 0110✓	(6,14) -110✗
(11) 1011✓	
(13) 1101✗	
(14) 1110✓	

Stage 2:

Minterm / Implicant	(0) 0000	(1) 0001	(3) 0011	(6) 0110	(8) 1000	(11) 1011	(13) 1101	(14) 1110
(0,1) 000-	x	x						
(0,8) -000	x				x			
(1,3) 00-1		x	x					
(3,11) -011			x			x		
(6,14) -110				x				x
(13) 1101							x	

EPI = {(0,8)-000, (3,11)-011, (6,14)-110, (13)1101}.
$\text{EPI}_{\text{Minterm}}$ = {(0)0000, (3)0011, (6)0110, (8)1000, (11)1011, (13)1101, (14)1110}.
Not all Minterms are covered.
$\overline{\text{EPI}}_{\text{Minterm}}$ = {(1)0001}
NEPI = {(0,1)000-, (1,3)00-1}.
Either of the two NEPI are valid to cover $\overline{\text{EPI}}_{\text{Minterm}}$, we can choose one of them.

The result is (0,1)000- and EPI: $F(A,B,C,D) = \overline{A}\overline{B}\overline{C} + \overline{B}\overline{C}\overline{D} + \overline{B}CD + BC\overline{D} + AB\overline{C}D$

3. Stage 1:

Minterms	Pass 1
(2) 0010✓	(2,3) 001-✗
(4) 0100✓	(2,6) 0-10✗
(8) 1000✓	(4,6) 01-0✗
(3) 0011✓	(4,12) -100✗
(6) 0110✓	(8,9) 100-✗
(9) 1001✓	(8,12) 1-00✗
(12) 1100✓	
(15) 1111✗	

Stage 2:

Minterm / Implicant	(2) 0010	(3) 0011	(4) 0100	(6) 0110	(8) 1000	(9) 1001	(12) 1100	(15) 1111
(2,3) 001-	x	x						
(2,6) 0-10	x			x				
(4,6) 01-0			x	x				
(4,12) -100			x				x	
(8,9) 100-					x	x		
(8,12) 1-00					x		x	
(15) 1111								x

EPI = {(2,3)001-, (8,9)100-, (15)1111}.
$\text{EPI}_{\text{Minterm}}$ = {(2)0010, (3)0011, (8)1000, (9)1001, (15)1111}. Not all Minterms are covered.
$\overline{\text{EPI}}_{\text{Minterm}}$ = {(4)0100, (6)0110, (12)1100}
NEPI = {(2,6)0-10, (4,6)01-0, (4,12)-100, (8,12)1-00}.
(4,6)01-0 et (4,12)-100 from NEPI are valid to cover $\overline{\text{EPI}}_{\text{Minterm}}$.

The result is (4,6)01-0, (4,12)-100 and EPI:
$F(A,B,C) = \overline{A}B\overline{D}+B\overline{C}\overline{D}+\overline{A}\overline{B}C+A\overline{B}\overline{C}+ABCD$