Sétif 1 University
Faculty of Sciences
Common Core Mathematics and Computer Science

1st semester (year 2025/26)
Machine Structures 1 course
Kara Abdelaziz professor

Machine Structures 1 exam solution

Exercise 1 : (4 points)

1. RAM: Random Access Memory. **(1 point)**

2. Hexadecimal symbols (10–15): The letters **A, B, C, D, E, and F**. **(1 point)**

3. Bits in a Byte: 8 bits. **(1 point)**

4. Result of A + 1 is: 1 **(1 point)**

Exercise 2 : (5 points)

1. Convert $(\mathbf{A2F})_{16}$ to octal going through binary: **(1 point)**

$(A\ 2\ F)_{16} = (1010 \mid 0010 \mid 1111)_2 = (101 \mid 000 \mid 101 \mid 111)_2 = (\mathbf{5\ 0\ 5\ 7})_8$

$(\mathbf{A2F})_{16} = (\mathbf{5057})_8$

2. Convert the fractional part $(\mathbf{0.4375})_{10}$ to hexadecimal using Successive Multiplication by 16: **(1 point)**

Multiplication	Integer part
0.4375 x 16 = 7.0	7

$(\mathbf{0.4375})_{10} = (\mathbf{0.7})_{16}$

3. Binary Addition: **(1 point)**

$(\mathbf{1101})_2 + (\mathbf{1011})_2 = (\mathbf{11000})_2$

```
 1 1 1 1
   1 1 0 1
+  1 0 1 1
----------
= 1 1 0 0 0
```

4. Convert $(\mathbf{E5})_{16}$ to decimal using Polynomial Formula with base-16: **(1 point)**

$(E \times 16^1) + (5 \times 16^0) = (14 \times 16) + (5 \times 1)$
$= 224 + 5 = (\mathbf{229})_{10}$

$(\mathbf{E5})_{16} = (\mathbf{229})_{10}$

5. The multiplication by the power of the base is a shift in the position of the fractional point: **(1 point)**

- **$(1011.01)_2 \times 2^3 = (1011.01)_2 \text{ x } (1000)_2 = (1011010)_2$**

Exercise 3 : (5 points)

1. $(-128)_{10} - (1)_{10}$ in 2C is $(-128)_{10} + (-1)_{10}$:

- Convert $(-128)_{10}$ to 8-bits Two's Complement by first converting $(+128)_{10}$ to binary using SED by 2. Knowing the number is negative, all the bits in the binary needs to be inverted to get 1C, then adding +1: **(0.5 point)**

Division	Remainder
128 ÷ 2 = 64	0
64 ÷ 2 = 32	0
32 ÷ 2 = 16	0
16 ÷ 2 = 8	0
8 ÷ 2 = 4	0
4 ÷ 2 = 2	0
2 ÷ 2 = 1	0
1 ÷ 2 = 0	1

$(+128)_{10} = (+10000000)_2$

negative number **(-)** → sign = 1
$(10000000)_2 \xrightarrow{1C} (01111111)_2 \xrightarrow{+1} (10000000)_2$
$(-128)_{10}$ = **$[10000000]_{2C\text{-}8}$**

- Convert $(-1)_{10}$ to 8-bits Two's Complement by first converting $(+1)_{10}$ to binary using SED by 2. Knowing the number is negative, all the bits in the binary needs to be inverted to get 1C, then adding +1: **(0.5 point)**

Division	Remainder
1 ÷ 2 = 0	1

$(+1)_{10} = (+1)_2$

negative number **(-)** → sign = 1
$(00000001)_2 \xrightarrow{1C} (11111110)_2 \xrightarrow{+1} (11111111)_2$
$(-1)_{10}$ = **$[11111111]_{2C\text{-}8}$**

- The addition $[10000000]_{2C\text{-}8} + [11111111]_{2C\text{-}8}$: **(0.5 point)**

$$\begin{array}{rl} & {}^{1}\,1\,0\,0\,0\,0\,0\,0\,0 \\ + & \;\;1\,1\,1\,1\,1\,1\,1\,1 \\ \hline = \not{1} | & 0\,1\,1\,1\,1\,1\,1\,1 \end{array}$$

$[10000000]_{2C\text{-}8} + [11111111]_{2C\text{-}8} = \mathbf{[01111111]_{2C\text{-}8}}$

- Decode $[01111111]_{2C\text{-}8}$ to Decimal, knowing sign is positive is to convert $(01111111)_2$ to Decimal using PF:

  sign = 0 → positive number **(+)**
  $(0x2^7) + (1x2^6) + (1x2^5) + (1x2^4) + (1x2^3) + (1x2^2) + (1x2^1) + (1x2^0) = \mathbf{(127)_{10}}$

$[01111111]_{2C\text{-}8} = \mathbf{(+127)_{10}}$

- Knowing the range of 2C 8-bits is [-128, +127], the subtraction should be $(-128)_{10} - (1)_{10} = \mathbf{(-129)_{10}}$ produces an **incorrect result**. The result don't belongs to the range, there is **an overflow**. **(0.5 point)**

2. Encoding the decimal $\mathbf{(-0.125)_{10}}$ to Single Precision 32-bit binary number:

- Converting the Number to Binary: **(0.5 point)**

  Convert the fractional part $(0.125)_{10}$ to a binary fraction using SM:

Multiplication	Integer part
0.125 x 2 = 0.25	0
0.25 x 2 = 0.5	0
0.5 x 2 = 1.0	1

$\mathbf{(0.125)_{10} = (0.001)_2}$

  Combine the integer and fractional parts: $\mathbf{(-0.125)_{10} = (-0.001)_2}$

- Determining the Sign bit (S): **(0.25 point)**

  The number -0.001 is negative → **S=1**

- Normalizing the Binary Number: **(0.25 point)**

  The binary number must be converted to the format $1.M\times2^{Ereal}$.
  $(-0.001)_2 = -1\times2^{-3}$
  Real Exponent: $\mathbf{E_r = -3}$
  The Mantissa: **M = 0**

- Calculating the Biased Exponent (E_b): **(0.5 point)**

  We have: **Bias = 127**.
  The Exponent formula:
  $E_r = E_b - Bias \rightarrow E_b = E_r + Bias$
  $E_b = -3 + 127 = \mathbf{(124)_{10}}$

Now, we convert E_b to its 8-bit binary representation using SED:

Division	Remainder
124 ÷ 2 = 62	0
62 ÷ 2 = 31	0
31 ÷ 2 = 15	1
15 ÷ 2 = 7	1
7 ÷ 2 = 3	1
3 ÷ 2 = 1	1
1 ÷ 2 = 0	1

$(124)_{10} = (01111100)_2$

E_b = [01111100]$_{8\text{-bits}}$

- Assembling the Final 32-bit Representation: **(0.5 point)**

We have the 3 fields in a 32-bits FP representation organized as follows:

Sing bit (1-bit) | Biased Exponent (8-bits) | Mantissa (23-bits)

Binary representation: **[1|011,1110,0000,0000,0000,0000,0000,0000]$_{FP\text{-}32}$**

In hexadecimal: **[BE000000]$_{FP\text{-}32}$**

3. Decoding the following ASCII sequence using the ASCII table: **(1 point)**

[48 65 6C 6C 6F 20 32 30 32 36]$_{ASCII}$

48: **H**	65: **e**	6C: **l**	6C: **l**	6F: **o**
20: **" "**	32: **2**	30: **0**	32: **2**	36: **6**

Decoded sentence: **"Hello 2026"**

Exercise 4: (6 points)

1. The Truth Table of the function F: **(1 point)**

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

The SoP form of the Truth Table is: $F(A,B,C) = \bar{A}BC + A\bar{B}C + AB\bar{C} + ABC$ **(1 point)**

2. Karnaugh map method using SoP: **(1 point)**

C \ AB	00	01	11	10
0	0	0	1	0
1	0	1	1	1

The SoP reduction is: $F(A,B,C,D) = AB + BC + AC$ **(1 point)**

3. Quine-MacCluskey method:

1. Stage 1: **(1 point)**

Minterms	**Pass 1**
(3) 011 ✓	(3,7) -11 ✗
(5) 101 ✓	(5,7) 1-1 ✗
(6) 110 ✓	(6,7) 11- ✗
(7) 111 ✓	

Stage 2: **(0.5 point)**

Implicant \ Minterm	(3) 011	(5) 101	(6) 110	(7) 111
(3,7) -11	x			x
(5,7) 1-1		x		x
(6,7) 11-			x	x

EPI = {(3,7) -11, (5,7) 1-1 , (6,7) 11-}.
$EPI_{Minterm}$ = {(3)011, (5)101, (6)110, (7)111}. all Minterms are covered.

The final result is $F(A,B,C) = BC + AC + AB$ **(0.5 point)**